

THE DISCOURSE OF ATTENDING TO PRECISION IN SECONDARY CLASSROOMS

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Attending to precision (ATP) is essential in mathematics. This study examined ATP instances through the lens of univocal (functioning to convey information) and dialogic (functioning to generate new meaning) discourse. Analysis of data from five secondary mathematics classrooms focused on whole-class instances of ATP with coding based on the univocal or dialogic nature of the discourse. Although instances were predominantly univocal, there was variation in whether the teacher's or student's idea was being transmitted. We share examples of the rare dialogic instances where the co-construction of meaning through discourse involved ATP in qualitatively different ways than the univocal instances.

Keywords: Classroom Discourse; Problem Solving; Metacognition; High School Education

Within mathematics, precision is highly valued because imprecision can lead to holes in arguments or faulty conclusions and miscommunication can prevent the development of shared meaning. As a steward of the discipline, then, school mathematics must help students learn to attend to precision and recognize that standards of precision are different within mathematical communities than they are in other communities. In the United States, the importance of attending to precision (ATP) was affirmed by its inclusion within the *Common Core State Standards for Mathematics* (2010). Not only is ATP a worthwhile end in its own right but also has the potential to support student learning. For example, by paying careful attention to the precise meaning of algebraic symbols, students can successfully transition from arithmetic reasoning to algebraic reasoning and be prepared for higher levels of mathematics (Kieran, 2007). In general, students, by interacting with teachers and classmates, can express and refine mathematical ideas together, constructing shared meanings and becoming legitimate participants in mathematical discourse (Lave & Wenger, 1991) rather than passive recipients of knowledge.

This study focused on ATP in secondary mathematics classrooms. Our goal was to see a broad range of instances of ATP and analyze the nature of the discourse in those instances within the whole-class public discourse. This work adds to the literature on ATP, which is relatively thin compared to well-developed topics such as problem solving or reasoning-and-proving.

Theoretical Framework

For this study we took a sociocultural perspective wherein student learning is viewed as intertwined with social interactions (Vygotsky, 1978) and used tools from discourse analysis to examine the interactions. In particular, we drew on Herbel-Eisenmann and Otten (2011) who framed the learning of a subject as the process of coming to participate meaningfully in the discourse of that subject's community. We consider ATP to be one of the characteristic practices of the mathematics classroom community as well as the broader mathematics community.

Discourse serves two important functions—to transmit an existing meaning from one person to another and to generate new meanings through the process of interacting (Bakhtin, 1986). The term *univocal* refers to discourse that is primarily intended to fulfill the transmission function, whereas *dialogic* refers to discourse that is primarily intended to generate new meaning. Mathematics classrooms in the United States predominantly feature univocal discourse (National Council of Teachers of Mathematics, 2014), yet various scholars have provided evidence for the value of dialogic discourse (e.g., Lobato, Clarke, & Ellis, 2005; Otten & Soria, 2014) and called for more research on benefits of different forms of classroom discourse (Howe & Abedin, 2013).

Research Question

This study addresses the following question: What is the univocal or dialogic nature of the discourse within instances of ATP in secondary mathematics classrooms? Although the question draws a binary distinction between univocal and dialogic discourse, we recognize that the underlying characteristics form a continuum (Truxaw & DeFranco, 2008) and that all discourse involves both deciphering meaning (univocal) and generating meaning (dialogic). Nonetheless, like others (Peressini & Knuth, 1998), we see practical value in distinguishing between discourse that is more prevalent in one than the other. This distinction is especially appropriate with regard to a mathematical practice such as ATP because scholars (e.g., Barwell & Kaiser, 2005) have pointed out that dialogic discourse is a practice-oriented process within a community.

Method

Setting and Participants

This study is part of a larger project focused on the mathematical practices of reasoning-and-proving, generalizing, and ATP. The project has two interrelated goals: (1) to support mathematics teachers in understanding and implementing these practices, and (2) to better understand how practicing teachers and students at various grade levels engage in the practices (or not) during classroom instruction. We worked with a group of teachers during their summer break to achieve goal 1, and we conducted classroom observations throughout the following academic year to achieve goal 2. This particular study reports on an analysis of ATP within the classroom observation data of the secondary teachers.

The project took place in a rural school district in the central United States. The district is predominantly white in ethnicity but comprises substantial economic diversity and diversity in parental education levels. The district performed slightly above the state average on the secondary mathematics standardized assessment.

Eight mathematics teachers volunteered to participate in the larger project, spanning grades 5–12. The teachers' backgrounds, teaching experiences, and philosophies toward mathematics education varied. For this study, we focused on five teachers (grades 8–12) and one focal class per teacher. We observed the focal classes on typical instructional days in the fall and winter. We intended to conduct three observations per class to capture a range of mathematical topics and gain a sense of the variability of discourse per classroom. When the discourse was limited, two observations were sufficient. When the discourse involved a range of interactions, we made three or four observations. This does not threaten our analysis since we do not seek to compare classes. Instead, we focus on the ATP instances when they arise and the nature of the discourse therein.

Data and Analysis

Classroom observations involved a single video camera and four digital audio recorders. Analysis began by flagging instances of ATP in the whole-class public discourse using a coding scheme based on the Common Core State Standards (2010), Koestler and colleagues (2013), and Fennell, Kobett, and Wray (2013, January). In particular, we looked for the following indicators:

- Emphasis, clarification, questions, or discussion in regard to
 - Defining terms or using them appropriately
 - Defining symbols or using them appropriately
 - Labeling units, graphs, or diagrams
 - Precision of calculations
 - Precision of measurements
 - Rounding or estimating

- Making or refining claims
- Giving explanations or justifications
- Appropriate mathematical precision within a non-mathematical problem context

Note that a teacher or student merely *being* precise did not necessarily result in a flag for ATP. More important was whether explicit *attention* was given to the precision. Having flagged the instances of ATP, we marked their beginning and end based on when the focus of the discourse shifted to and from the issue of precision and then transcribed each instance. We compiled durations and descriptive statistics for each class and for the set of ATP instances overall.

Next, we coded each instance of ATP based on the distinction between *univocal* and *dialogic* discourse. Building on the definitions above, we operationalized these constructs using the following characteristics, adapted from Wegerif (2006) and Truxaw and DeFranco (2008):

- Univocal
 - transmission of meaning (giver and receiver(s))
 - closed discourse (answer or information concludes the exchange)
- Dialogic
 - shared development of meaning (give-and-take)
 - open discourse (information or ideas spur further discourse)

Note that having multiple speakers does not imply dialogic interaction. Multiple speakers can be involved in transmitting meaning rather than in opening up a discourse space for making new meaning. Also, a single turn, such as a teacher's question, cannot be coded on its own because the potential to spur dialogic discourse does not necessarily do so. Thus, we considered interactions overall when making coding decisions. In rare cases, there was difficulty coding but these were discussed until consensus was reached.

Our final phase of analysis was to look across the coding for patterns and themes both within and across the univocal and dialogic instances. These themes were used to structure the Findings, presented below with several examples used to illustrate the themes.

Findings

We found 140 instances of ATP in the classroom data, with an average duration of around 40 seconds per instance. Some were very brief and others lasted a few minutes. Overall, univocal instances (131) were more common than dialogic instances (9). Only two classes exhibited dialogic instances and univocal ATP was still predominant. Below we illustrate some themes that emerged from the univocal instances of ATP and give examples of the rare dialogic instances.

Univocal Instances of Attending to Precision

The 131 instances of univocal ATP comprised 93.6% of the total instances and ranged in duration from 3 seconds to 2 minutes, 48 seconds. The nature of the ATP in these instances varied. In some cases, teachers briefly pointed out a lack of precision, a need to be precise, or prompted students to use precise terminology. In other cases, teachers and students attended to precision of their communication to assure that the transmission of meaning was successful.

Many instances of univocal ATP were relatively brief, involving a teacher making an explicit remark about precision. For example, when working with graphs, Mr. Forrest said, "[The graph] doesn't have to be perfect, but I'm going to make sure I put the x -intercepts about where they are on this graph and the y -intercept about where it is on that graph." These instances were univocal because they simply involved a teacher transmitting an idea or warning with regard to ATP.

Other brief instances of univocal ATP involved teachers pressing for precise terminology. For example, Ms. Finley was working with students to prove that two triangles were congruent. One of the steps in the proof involved side NG being shared by both triangles.

Finley: OK, GN is equal to NG, which is true. My question to you is, why?

Students: Same line?

Finley: It is the same line, but we have a name for that... It's a property that we did back in Chapter 2, that seemed really silly at the time, but...

Students: Reflexive!

Finley: Reflexive. OK?

Male Student: I said it. I knew it.

Ms. Finley prompted the class to use the precise term in the written proof rather than simply saying that GN and NG are the “same line” segment. This interaction was univocal because the teacher knew the property and terminology she wanted and the interaction ended as soon as the term was supplied. It was not opened up into a discussion of the reflexive property itself or to other imprecisions such as the conflation of line and line segment. The student's final turn confirms that this interaction was univocal, directed toward supplying known information.

Longer instances of univocal ATP most often involved the teacher and students engaging in discourse to clarify or correct the communication of a mathematical idea. These instances were univocal because they were, in essence, a transmission of someone's idea and ATP played a role in assuring that the meaning was transmitted and received as intended. For instance, Ms. Finley drew a pair of triangles on the board (see Figure 1) and claimed they were congruent, but she purposefully wrote the congruence statement with the vertices listed in an incorrect order.

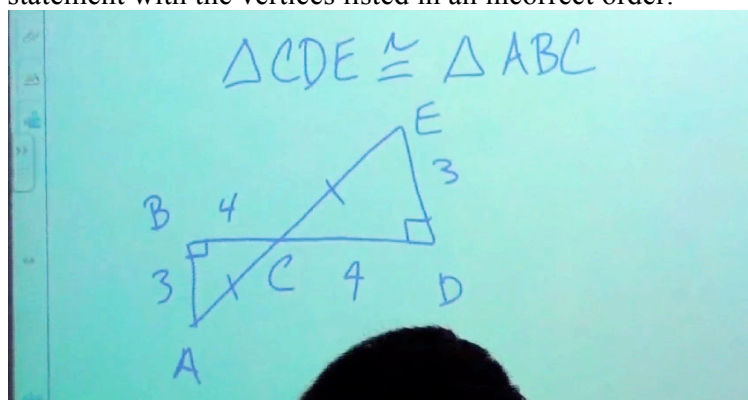


Figure 1: An incorrect statement about congruent triangles

Finley: OK, so, I'm telling you I'm wrong. Emily thinks I'm wrong because I can't draw. I get that. The question is, why am I really wrong?

Female Student: Does it have something to do with the right angle?

Male Student 1: You don't have congruent angles.

Students: Yeah.

Ms. Finley guided the students through a quick verification that, in fact, all the corresponding sides and angles were congruent to one another, so the triangles were truly congruent.

Finley: So there's something else that must be wrong with my thinking. Melissa?

Melissa: The numbers are wrong, or the letters.

Finley: The letters are in the wrong order. What did I not match up right?

Male Student 2: Ohhh, I see.

Finley: C should match with...

Students: C.

Finley: C [instead of A]. So, this [congruence statement] is not true, and don't make this mistake on your homework because if you do, we'll have a problem.

Note that there was not only ATP in this interaction but also a suggestion from Ms. Finley to be precise in the future. This interaction was univocal because Ms. Finley had a particular error she wanted to discuss and the exchange concluded when it was identified. Students made bids to open up a dialogic exchange when they questioned the angles instead of the labeling, but this did not lead to dialogic discourse as Ms. Finley led students through verifying the corresponding parts of the triangles were indeed congruent and then proceeded with the ATP of the labeling.

Less often, the long instances of univocal ATP focused on students' ideas. The following instance is from Mr. Forrest's lesson on graphs of polynomials. Mr. Forrest asked the class about "tangent to" and the interaction then focused on clarifying a student's idea about the phrase.

Forrest: Is the graph tangent to the x -axis, or does the graph continue through the x -axis at each x -intercept? ... Does anybody know what "tangent to" is?

Dustin: Stops at it.

Forrest: How do you mean, "Stops at it"?

Dustin: Like, whenever it goes down (drawing in the air) and touches it [the x -axis], it immediately goes right back up and that's either the furthest point up or the furthest point down in that part.

Forrest: Alright, I like your explanation Dustin. Could you draw a picture on the board? [Dustin draws a curve on the board; see Figure 2]



Figure 2: Dustin's example of a curve tangent to the x -axis.

ATP occurred as Mr. Forrest asked Dustin to clarify his words, "Stops at it", and to represent his idea graphically. Although Mr. Forrest prompted the ATP, Dustin's idea was the focus. The interaction was univocal because it involved clarifying Dustin's transmission to the class.

Dialogic Instances of Attending to Precision

Although dialogic instances of ATP were rare, they are important to consider because they illustrate a different form of interaction and engagement with ATP. All 9 dialogic instances involved precision of language or communication, whether it be constructing definitions, refining student conjectures, or building upon (rather than merely transmitting) a student's idea.

The following excerpt is from Mr. Forrest's class as they discussed the graph of $y = \sqrt{-x}$. Mr. Forrest asked a small group how the calculator was able to display a graph even though the square root of a negative number is imaginary. He then engaged the whole class in this idea.

Forrest: Can anybody explain why you still got a graph there? What do you think, Matt?

Matt: Because all the x -values are negative, it's gonna make it positive.

Forrest: So, “Because all the x -values are negative, it’s gonna make them...”

Matt: Like, the x -values in the graph, whenever you plug them in it’s gonna invert them into a positive number.

Forrest: Alright, so can you give me an example? Because we got [writes on board] y equals the square-root of negative x , right?

Matt: So x is negative three.

Forrest: So if we made an xy -table, plugged in negative three?

Matt: Yeah.

Forrest: We get the square root of...

Matt: Three.

Forrest: Three, right? Because, if you think of it another way, wouldn’t that be the square root of negative negative three?

Female Student: Uh huh.

Forrest: Which is the square root of negative negative 3. This is why it’s possible. Because even though it’s the square root of negative x , what kind of values can we plug in for x ? We can plug in negative values for x to give the square root of x . Does that make sense?

Matt offered an idea and the subsequent discourse involved attending to the precision of his language to facilitate communication while also building new meaning. For example, Mr. Forrest repeated Matt’s original statement, which contained two vague referents (“it’s” and “it”), then Matt added to his explanation. Mr. Forrest also requested an example and said, “So if we made an xy -table, plugged in negative three?” to push Matt to express his idea more clearly. Matt was able to give a specific example and Mr. Forrest then built on that idea to move toward a more general idea based on that shared meaning. It is important to note that Mr. Forrest was not the one doing all the reasoning. He did contribute, but he also pushed Matt to illustrate the reasoning behind his idea, which allowed Mr. Forrest to build upon and extend Matt’s original reasoning.

This second example is also from Mr. Forrest’s class because his class exhibited nearly all the dialogic instances of ATP. Here, Mr. Forrest was reviewing the definition of a polynomial in a univocal manner when dialogic discourse took place around the definition of whole number.

Forrest: Polynomial functions have only whole number exponents on the variables. What are whole numbers?

MS: Like, number one would be a whole number.

Forrest: That’s true. Specifically, what is the set of whole numbers? ... [pause] Because when we say whole numbers, it’s actually a very specific mathematical set, right?

Zack: One, two, three, four.

Forrest: Say it again.

Zack: I just counted them.

Forrest: OK.

Ben: Like zero to infinity, positive.

Forrest: Zero to positive infinity?

Ben: Mm hmm.

Male Student: Wait, is there other kinds of infinity?

Henry: It could be fractions and decimals, and those aren’t.

Forrest: Alright, so if we say zero to positive infinity, we’re including too much? Is that what you’re saying, Henry? [Henry nods] OK. So be specific.

Dustin: Starting at one and counting up by ones.

Forrest: Starting at one and counting up by ones. Is that the whole numbers?

Female Student: Sure.

Forrest: Sure.

Amy: It doesn't have zero.

Male Student: No, zero isn't.

Forrest: I don't really like pestering anyone, but I see lots of college algebra books right in front of people. You could find out, right?

The precision of language is apparent as Mr. Forrest repeatedly called on students to clarify how they describe whole numbers. In trying to precisely describe the set of whole numbers, several students contributed ideas in a dialogic manner. One student gave an example of a whole number ("one"), Zack then gave a pattern of whole numbers ("one, two, three, four"), and Ben attempted a definition ("zero to infinity, positive"). Then, Henry pointed out that Ben's definition included too many numbers. Mr. Forrest clarified this idea but then did not refine the definition. Instead, he left the discourse open and Dustin then provided a new definition ("starting with one and counting by ones"). Mr. Forrest and other students weighed in, unsure of whether this was an acceptable definition. At that point, Mr. Forrest directed attention toward the textbook definition, which ended the dialogic ATP. Although this interaction was dialogic, all shared ideas were not taken up in the discourse. In particular, one male student raised a question about "kinds of infinity" that was not addressed, and Amy's comment about zero possibly being a whole number was not discussed but was circumvented by appealing to the textbook definition. An exploration and determination with regard to zero would have been a further opportunity for ATP.

Discussion

We examined instances of ATP in the whole-class discourse in five secondary mathematics classrooms and focused on the univocal or dialogic nature of those instances. Some ATP instances were brief—for example, a teacher pointing out the need for labels in a graph or calling for the use of a specific term. Other instances were longer interactions focused on clarifying the transmission of someone's idea, many times the teacher's but sometimes a student's. Overall, the vast majority (93.6%) of ATP instances were univocal. Although past research (e.g., Stigler & Hiebert, 1999) led us to expect univocal discourse, its dominance was surprising for two reasons. First, due to grade level, topic, and teacher background variability, we expected at least some of the classrooms to exhibit more substantial dialogic discourse. As it turned out, even the class with the most dialogic instances (Mr. Forrest's) still had nearly 90% univocal. Second, many of the illustrations of the role that ATP can play in mathematics education (Koestler et al., 2013) involve dynamic, dialogic interactions wherein meaning is constructed through a collaborative process of critique, refinement, and extension. Such instances of ATP were rare in our data.

When dialogic ATP did occur, the meaning-making, by definition, was qualitatively different than in univocal interactions. A key question, then, is what spurred the dialogic discourse? The answer is certainly a multitude of factors and would require further analysis to parse out, but we can speculate that it was a confluence of the mathematical content or task available as a focus for the discourse, the teacher's discourse moves, and the students' contributions to the discourse. It is also important to consider ATP in particular, because it is possible that certain aspects of ATP—attention to labels, technical terms, etc.—lend themselves to univocal discourse because they are somewhat normative. Other aspects of ATP—the process of defining, clarifying reasoning—may naturally be better suited for dialogic discourse. This study provides preliminary evidence that such may be the case, but further research is needed.

Another important contribution from this study has to do with the role of students within ATP discourse. Many scholars have brought attention to the value of engaging students in dialogic discourse (e.g., Lobato et al., 2005; Otten & Soria, 2014). The present study highlights that involving students actively in the classroom discourse is not necessarily the same as engaging students in *dialogic* discourse. As we saw, even when students' ideas were the focus of ATP interactions, it often

remained univocal because the purpose was to transmit or clarify those ideas to the rest of the class, which is distinct from engaging students in the collaborative meaning-making of dialogic discourse. Heeding Clarke's (2006) warning, we do not intend to set this as an unproductive dichotomy between univocal and dialogic discourse or of teacher-focused and student-focused discourse. Rather, this is meant to be a call to attend with precision to the different experiences provided to students in mathematics classrooms with regard to ATP.

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References

- Bakhtin, M. (1986). *Speech genres and other late essays*. Austin, TX: University of Texas.
- Barwell, R., & Kaiser, G. (2005). Mathematics education in culturally diverse classrooms. *ZDM - The International Journal on Mathematics Education*, 37, 61-63.
- Clarke, D. (2006). Using international research to contest prevalent oppositional dichotomies. *ZDM - The International Journal on Mathematics Education*, 38, 376-387.
- Fennell, F., Kobett, B., & Wray, J. (2013, January). *Using look for's to consider the Common Core State Standards*. Paper presented at the annual meeting of the Association of Mathematics Teacher Education, Orlando, FL.
- Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education*, 42, 451-485.
- Howe, C., & Abedin, M. (2013). Classroom dialogue: A systematic review across four decades of research. *Cambridge Journal of Education*, 43, 325-356. doi: 10.1080/0305764X.2013.786024
- Kieran, C. (2007). Learning and teaching of algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707-762). Charlotte, NC: Information Age Publishing.
- Koestler, C., Felton, M. D., Bieda, K. N., & Otten, S. (2013). *Connecting the NCTM Process Standards and the CCSSM Practices*. Reston, VA: National Council of Teachers of Mathematics.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, England: Cambridge University Press.
- Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education*, 36, 101-136.
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all*. Reston, VA: Author.
- National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.
- Otten, S., & Soria, V. M. (2014). Relationships between students' learning and their participation during enactment of middle school algebra tasks. *ZDM - The International Journal on Mathematics Education*, 46, 815-827. doi: 10.1007/s11858-014-0572-4
- Peressini, D. D., & Knuth, E. J. (1998). Why are you talking when you could be listening? The role of discourse and reflection in the professional development of a secondary mathematics teacher. *Teaching and Teacher Education*, 14, 107-125.
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Simon and Schuster.
- Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education*, 39, 489-525.
- Vygotsky, L. (1978). *Mind in Society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Wegerif, R. (2006). Dialogic education: What is it and why do we need it? *Education Review*, 19(2), 58-66.